9.10. Lift and Drag on Immersed Objects

The force acting on an object immersed in a fluid flow is comprised of the force due to pressure over the surface and the force due to viscous wall shear stresses (Figure 9.25). If we know the pressure (p) and shear stress (τ) distribution over the object, then,

$$F_{p,i} = \int_A -p dA n_i, \tag{9.192}$$

$$F_{s,i} = \int_A \tau_{ji} dA n_i. \tag{9.193}$$

where F_p is the force due to the pressure component, F_s is the force due to the shear stress component, A is the surface area of the object, and n_i are the unit normal vector components of the differential surface area. The component of the resultant force acting in the direction *parallel to the incoming flow* is known as the drag force, F_D, and the component *perpendicular to the incoming flow* is known as the lift force, F_L.

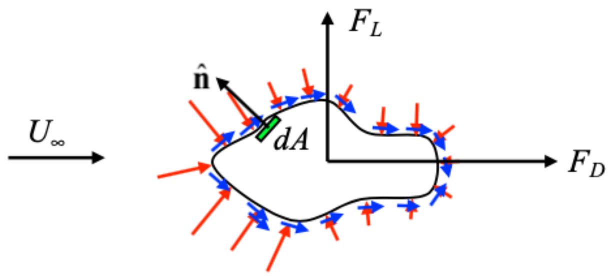


Figure 9.25. An illustration of the pressure and shear force distributions over an immersed object. The resultant lift F_L and F_D forces acting on the object are also shown.

Notes:

(1) The pressure force component of the drag is known as the form drag while the shear stress drag component is known as the skin friction drag.
(2) A streamlined body is one in which the (skin friction drag) $\gg$ (form drag) (refer to Figure 9.26).

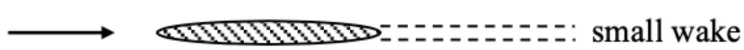


Figure 9.26. An example of a streamlined body.

A bluff body is one in which the (form drag) $\gg$ (skin friction drag) (refer to Figure 9.27).

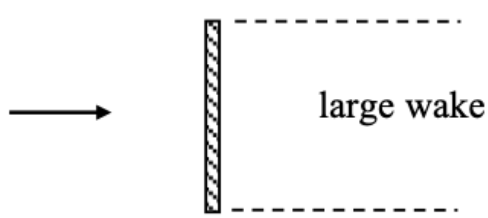


Figure 9.27. An example of a bluff body.

(3) The lift and drag are often expressed in dimensionless form as lift and drag coefficients, C_L and C_D,

$$C_L := \frac{L}{\frac{1}{2}\rho U_\infty^2 A} \quad \text{and} \quad C_D := \frac{D}{\frac{1}{2}\rho U_\infty^2 A}, \tag{9.194}$$

where A is usually the frontal projected area, i.e., the area seen from the front of the object, for a bluff body, or the planform area, i.e., the area seen from above, for a streamlined body (Figure 9.28). To avoid ambiguity, it is best to report what area is used to form the lift and drag coefficients.

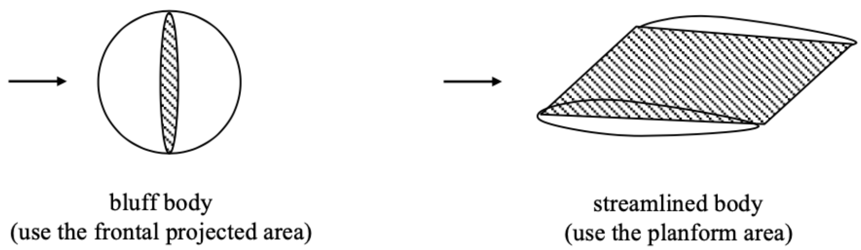


Figure 9.28. Illustrations of the frontal projected area (left) and the planform area (right).

9.10.1. Flow around a Sphere at Different Reynolds Numbers

Since flow around spheres is common in practice, it's worthwhile to examine the flow behavior around a sphere at different Reynolds numbers,

$$\mathrm{Re}_D = \frac{U_\infty D}{\nu}, \tag{9.195}$$

where U_∞ is the upstream flow speed, D is the sphere diameter, and ν is the fluid's kinematic viscosity. Although we'll specifically look at flow around a sphere, the general patterns shown here are observed for many other objects too, although the details may be different.

Figure 9.29 shows sketches of different flow regimes as a function of Reynolds number while Figure 9.30 shows corresponding photographs. At the smallest Reynolds number the flow streamlines are symmetric between the front and back halves of the sphere since fluid inertia is negligible. This regime is known as the Stokes or creeping flow regime. As the Reynolds number increases, inertia becomes more significant and a wake with fixed eddies forms downstream of the sphere. At larger Reynolds numbers, the eddies no longer remain attached behind the sphere and, instead, detach periodically from the sphere and are carried downstream. This phenomenon is known as a Kármán Vortex Street. Figure 9.31 shows striking photographs of Kármán vortex streets. At even larger Reynolds numbers the flow structure around the sphere becomes even more complex. A laminar boundary layer forms on the front half of the sphere and then separates at an angle of approximately 80° from the leading stagnation point. A laminar wake forms near the sphere downstream surface, which transitions to turbulence further downstream. At a Reynolds number of approximately 200,000 the flow undergoes what's referred to as the drag crisis. At this Reynolds number the boundary layer transitions from laminar to turbulent on the sphere surface and, thus, separates further downstream. The form drag on the sphere, which is the primary contributor to the overall drag, decreases as a result and the drag coefficient drops significantly. This decrease in drag coefficient is discussed further in the following notes. Reynolds numbers larger than 200,000 result in a nearly fully turbulent boundary layer.

Notes:

(1) The periodic shedding of vortices off an object results in periodic forces exerted on the object in the spanwise direction. The Tacoma Narrows bridge disaster from 1940 (Figure 9.32) occurred because a structural natural frequency of the bridge matched the frequency of the shedding vortices, causing the bridge to resonant and eventually collapse.

(2) Experimental measurements have shown that the dimensionless frequency of the shedding vortices, f, expressed as a Strouhal number, i.e., $\text{St} = fD/V$, remains relatively constant at 0.2 for $100 < \text{Re}_D < 1 \times 10^6$. The fact that the Strouhal number is insensitive to the Reynolds number over a wide range of Reynolds numbers has been used to design a type of flow velocity meter known as a vortex flow meter (Figure 9.34). By measuring the frequency of the forces acting on the obstruction (of known size) and knowing that the Strouhal number is approximately equal to 0.2, the flow velocity can be estimated.

(3) The drag coefficient acting on a sphere is shown in Figure 9.35. Commonly-used curve fits for the drag coefficient are,

$$\text{Re}_D < 1: \quad C_D = \frac{24}{\text{Re}_D} \quad \text{(Stokes' drag law)}, \tag{9.196}$$

$$\text{Re}_D < 5: \quad C_D = \frac{24}{\text{Re}_D}\left(1 + \frac{3}{16}\text{Re}_D\right) \quad \text{(Oseen's approximation)}, \tag{9.197}$$

$$0 \leq \text{Re}_D \leq 2 \times 10^5: \quad C_D = \frac{24}{\text{Re}_D} + \frac{6}{\left(1 + \sqrt{\text{Re}_D}\right)} + 0.4, \tag{9.198}$$

$$0 \leq \text{Re}_D < 2 \times 10^5: \quad C_D = 0.44 \quad \text{(Newton's Law)}. \tag{9.199}$$

The only analytically-derived expression for the drag coefficient is for the Stokes flow regime and Oseen's approximation. The remainder of the drag coefficient relations are empirical curve fits. Note the abrupt decrease in the drag coefficient at the drag crisis, which occurs at a Reynolds number of 200,000. This is where the boundary layer transitions to turbulence, delaying separation and decreasing the form drag. The onset of the drag crisis is dependent on the surface roughness (Figure 9.36). Increasing roughness causes the transition to turbulence to occur sooner and moves the drag crisis to a smaller Reynolds number. The dimples on a golf ball serve this same purpose. By decreasing the drag coefficient on the ball, the ball will travel further (and make golf ball manufacturers more money!).

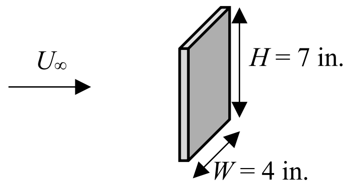
Notes:

(a) Interestingly, the Reynolds numbers for a 95 mph baseball, a 170 mph golf ball, a 100 mph cricket ball, and a 140 mph tennis ball are all near the drag crisis.

(4) Drag coefficients for irregular shapes are usually found experimentally or, in some cases, computationally. Figures 9.37 and 9.38 give drag coefficients for a variety of objects.

Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. What would the force be if you held your hand out of the window of a jet flying at 550 mph?

SOLUTION:
Model your hand as a rectangular flat plate oriented normal to the flow as shown in the following figure.



The drag force on your hand (the plate) is,

$$D = c_D \frac{1}{2}\rho U_\infty^2 A, \quad (1)$$

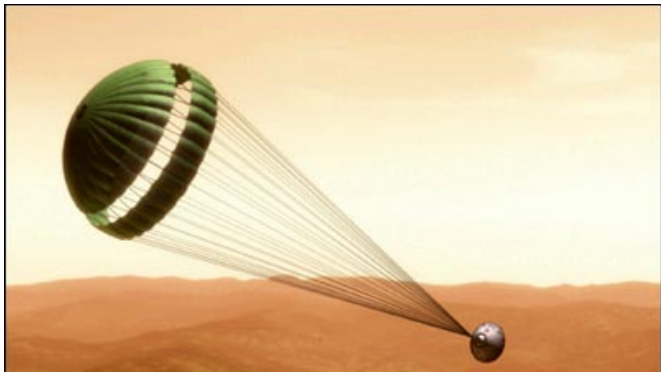
where,

$\rho_{air} = 2.38*10^{-3}$ slug/ft^3,
$\mu_{air} = 3.737*10^{-7}$ slug/(ft.s),
$U_{\infty,1}$ = 55 mph = 80.7 ft/s,
$U_{\infty,2}$ = 550 mph = 807 ft/s,
$A = WH$ = (4 in.)(7 in.) = 28 in.^{2} = 0.19 ft^2,
$c_{D,\text{flat plate}} \approx 1.2$ (obtained from a drag coefficient table for a flat plate),
for $H/W = 1.75$,
$\text{Re}_{Dh} = U_\infty D_h/\nu$ (Reynolds number based on a hydraulic diameter),
$D_h = 4LW/[2(W+H)]$ = 5.1 in. = 0.42 ft (hydraulic diameter),
=> $\text{Re}_{Dh,1}$ = 217,000, $\text{Re}_{Dh,2}$ = 2,170,000 (these values aren't used in the calculation other than to ensure the drag coefficient in the table is in the correct range),

=> $\boxed{D = 1.80 \text{ lb}_f \text{ at 55 mph}}$ and $\boxed{D = 180 \text{ lb}_f \text{ at 550 mph}}$.

Note that at 550 mph compressibility of the air would be significant and should be included in the drag calculations. Furthermore, the upstream air density would be smaller than the sea level value since the jet would be at an elevation higher than sea level. Thus, the drag estimate at 550 mph is questionable.

A parachute was used during part of the landing sequence to deposit the *Spirit* rover on the Martian surface. The parachute had a fully-open, projected diameter of 14.1 m and was designed to slow the landing package (lander and rover) to a terminal speed of 65 m/s (retro-rockets were used to bring the landing package to a near zero vertical velocity). If the mass of the landing package was 544 kg, what was the drag coefficient for the parachute? Assume the gravitational acceleration on Mars is 3.72 m/s^2 and that the density of the Martian atmosphere near the surface is 0.016 kg/m^3.



SOLUTION:

At terminal speed, the weight of the landing package must be balanced by the drag acting on the parachute (neglecting the drag on the landing package itself),

$$\sum F_y = 0 = D - W, \tag{1}$$

where,

$$D = c_D \tfrac{1}{2}\rho V_T^2 A, \tag{2}$$

$$W = mg. \tag{3}$$

Substitute and re-arrange to solve for the drag coefficient,

$$c_D \tfrac{1}{2}\rho V_T^2 A - mg = 0 \tag{4}$$

$$\boxed{c_D = \frac{mg}{\tfrac{1}{2}\rho V_T^2 A}} \tag{5}$$

Using the given data,

m = 544 kg
g = 3.72 m/s^2
ρ = 0.016 kg/m^3
V_T = 65 m/s
A = 156.1 m^2 (=π/4*(14.1 m)^2)
$\Rightarrow$ $\boxed{c_D = 0.38}$

In the book/movie *The Martian*, the mission of a crew of astronauts is derailed by a massive Martian windstorm. If the Martian atmosphere has a density of 0.016 kg/m^3 and the wind speed is 26.8 m/s (= 60 mph), what is the drag force acting on astronaut Mark Watney? Based on wind tunnel testing, assume that the drag coefficient multiplied by the frontal projected area of a typical person is $C_D A$ = 0.84 m^2 (see, for example, Table 7.3 in White, F.M., *Fluid Mechanics*, 7th ed., McGraw-Hill).

What wind speed on Earth would produce an equivalent drag force?

SOLUTION:
The drag force is given by,

$$D = C_D \tfrac{1}{2}\rho_{\text{Mars}} V^2 A\,, \tag{1}$$

where,

$C_D A$ = 0.84 m^2 (given),
ρ_{Mars} = 0.016 kg/m^3,
V = 26.8 m/s,
=> $\boxed{D = 4.8 \text{ N } (= 1.1 \text{ lb}_f)}$

Thus, we see that the author took considerable artistic liberty in portraying the damage caused by a Martian windstorm.

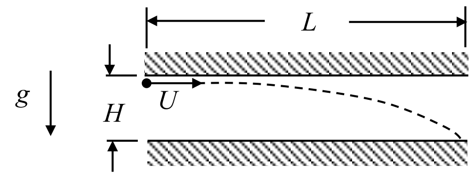
To determine the wind speed on Earth that would cause the same drag force, set the drag forces for Mars and Earth equal,

$$C_D \tfrac{1}{2}\rho_{\text{Mars}} V_{\text{Mars}}^2 A = C_D \tfrac{1}{2}\rho_{\text{Earth}} V_{\text{Earth}}^2 A\,, \tag{2}$$

$$V_{\text{Earth}} = V_{\text{Mars}}\sqrt{\frac{\rho_{\text{Mars}}}{\rho_{\text{Earth}}}}\,. \tag{3}$$

Using ρ_{Earth} = 1.23 kg/m^3, $\boxed{V_{\text{Earth}} = 3.1 \text{ m/s } (= 6.8 \text{ mph})}$, which corresponds to a light breeze.

Gravity settling tanks are sometimes used to separate particles from a fluid stream. Estimate the critical length, L, for capturing a particle by gravity settling in the channel shown below. Express your answer in terms of the channel height, H, the fluid velocity, U, the fluid density, ρ_f, the fluid dynamic viscosity, μ_f, particle density, ρ_p, the particle diameter, d, and the acceleration due to gravity, g. You may assume that the particle diameter is very small and that the fluid velocity profile in the channel is uniform. How will the length L change if the particle diameter is doubled?



SOLUTION:

In order to capture the particle, we want the particle to settle on the base before passing through the device, i.e.,

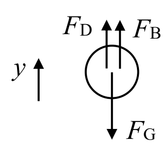
$t_{\text{settling}} < t_{\text{residence}}$

where,

$t_{\text{residence time}} = L/U$ (chamber length / fluid velocity)

$t_{\text{settling time}} = H/U_{\text{pt}}$ (settling height / particle terminal velocity)

The particle terminal velocity can be determined by considering a free body diagram acting on the particle. The forces acting on the particle include a drag force, F_D, a buoyant force, F_B, and a gravitational force, F_G.



$$\sum F_y = 0 = F_D + F_B - F_G$$

$$0 = 3\pi\mu_f U_{pt} d + \rho_f \frac{\pi}{6} d^3 g - \rho_p \frac{\pi}{6} d^3 g$$

Note that Stokes drag has been assumed for the particle drag force since the particle Reynolds number is assumed to be very small. Solving the previous equation for the terminal velocity, U_{pt}, gives,

$$U_{pt} = \frac{\left(\rho_s - \rho_f\right) g d^2}{18\mu_f}$$

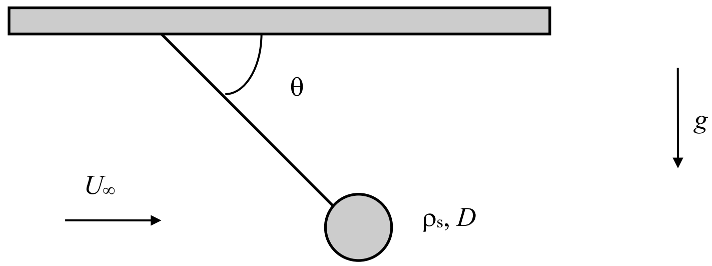
Since the settling time must be less than the residence time,

$$\frac{L}{U} > \frac{18\mu_f H}{\left(\rho_s - \rho_f\right) g d^2} \Rightarrow \boxed{\frac{L}{H} > \frac{18\mu_f U}{\left(\rho_s - \rho_f\right) g d^2}}$$

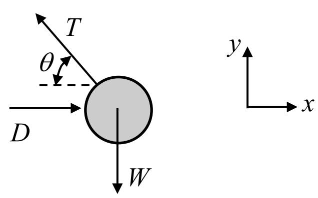
The length of the settling chamber, L, will decrease by a factor of four if the particle size doubles.

A heavy sphere attached to a string will hang at an angle, θ, when immersed in a stream of velocity U_∞ as shown in the figure.

a. Derive an expression for θ as a function of the sphere and flow properties.
b. What is θ if the sphere is steel (SG=7.86) of diameter 3 cm and the flow is sea-level standard air at U_∞ = 40 m/s? Neglect the string drag.
c. For the same parameters as in part (b), at what velocity will the angle be 45°?



SOLUTION:
Draw a free body diagram for the sphere and balance forces in the vertical and horizontal directions.



$$\sum F_y = 0 = T\sin\theta - W \;\Rightarrow\; T = \frac{W}{\sin\theta} = \frac{mg}{\sin\theta} \tag{1}$$

$$\sum F_x = 0 = -T\cos\theta + D \;\Rightarrow\; T = \frac{D}{\cos\theta} = \frac{c_D \frac{1}{2}\rho_a U_\infty^2 \frac{\pi}{4} d^2}{\cos\theta} \tag{2}$$

Set the tensions equal in Eqs. (1) and (2) and simplify.

$$\frac{mg}{\sin\theta} = \frac{c_D \frac{1}{2}\rho U_\infty^2 \frac{\pi}{4} d^2}{\cos\theta} \;\Rightarrow\; \tan\theta = \frac{mg}{c_D \frac{1}{2}\rho_a U_\infty^2 \frac{\pi}{4} d^2} = \frac{\rho_S \frac{\pi}{6} d^3 g}{c_D \frac{1}{2}\rho_a U_\infty^2 \frac{\pi}{4} d^2} \tag{3}$$

$$\boxed{\therefore \tan\theta = \frac{4}{3}\frac{1}{c_D}\left(\frac{\rho_S}{\rho_a}\right)\left(\frac{gd}{U_\infty^2}\right)} \tag{4}$$

where the drag coefficient, c_D, is a function of the Reynolds number based on the sphere diameter, *i.e.*, $\mathrm{Re}_d = U_\infty d/\nu_a$.

For the given data,

SG = 7.86 $\Rightarrow$ ρ_S = 7860 kg/m^3
U_∞ = 40 m/s
d = 0.03m
ρ_a = 1.23 kg/m^3
g = 9.81 m/s^2
ν_a = 1.1*10^{-5} m^2/s
$\Rightarrow$ Re_d = 110,000 $\Rightarrow$ c_D = 0.44
$\Rightarrow$ $\boxed{\theta = 74°}$

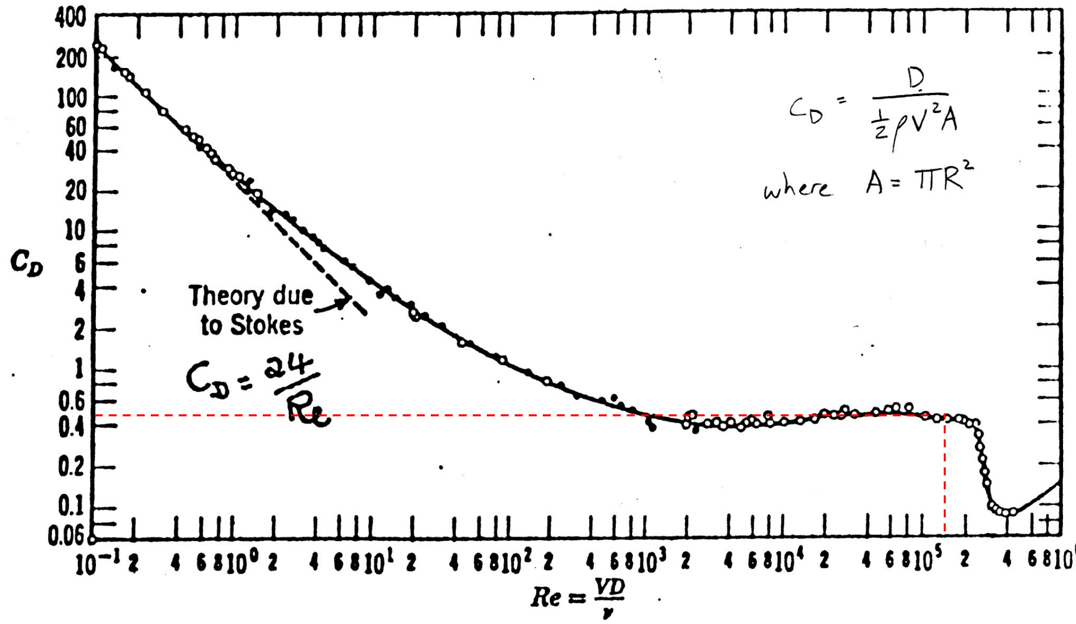


Fig. 8.32 Drag coefficient of a sphere as a function of Reynolds number (Ref. 13).

To find the wind speed corresponding to the given angle, we need to iterate to a solution since the drag coefficient is a complex function of the flow speed. The following algorithm can be used for iteration. Note that other algorithms may also be possible.

1. Guess a value for the speed $U_{\infty,\text{guess}}$.
2. Calculate the Reynolds number, $\text{Re} = U_\infty d/\nu$.
3. Use the plot shown above to determine the drag coefficient, c_D.
4. Calculate the speed $U_{\infty,\text{calc}}$ using a re-arranged Eq. (4) and the given angle,

$$U_\infty = \sqrt{\frac{4}{3}\frac{1}{c_D}\frac{gd}{\tan\theta}\left(\frac{\rho_S}{\rho_a}\right)}. \tag{5}$$

5. If $U_{\infty,\text{calc}} = U_{\infty,\text{guess}}$ (to within some acceptable tolerance), then stop the iterations because the solution has been found. If $U_{\infty,\text{calc}} \neq U_{\infty,\text{guess}}$, then let $U_{\infty,\text{guess}} = U_{\infty,\text{calc}}$ and repeat steps 2 – 5.

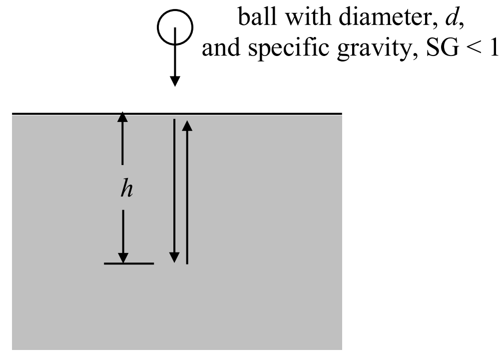
For example, starting with $U_{\infty,\text{guess}}$ = 1.0 m/s.

ρ_s [kg/m^3] =	7860
d [m] =	0.03
ρ_a [kg/m$_3$] =	1.23
g [m/s^2] =	9.81
ν_a [m$_2$/s] =	0.000011
θ [deg] =	45

$U_{inf,guess}$ [m/s]	Re [-]	c_D [-]	$U_{inf,calc}$ [m/s]
1.00	2727	0.42	77.21
77.21	210578	0.40	79.51
79.51	216841	0.39	80.68
80.68	220025	0.38	81.36
81.36	221890	0.37	81.79
81.79	223065	0.37	82.07
82.07	223837	0.37	82.26
82.26	224357	0.37	82.40
82.40	224715	0.37	82.49
82.49	224964	0.37	82.55
82.55	225138	0.37	82.60
82.60	225261	0.37	82.63
82.63	225348	0.37	82.65
82.65	225410	0.37	82.67
82.67	225453	0.37	82.68
82.68	225485	0.37	82.69
82.69	225507	0.37	82.69
82.69	225523	0.37	82.70
82.70	225534	0.37	82.70

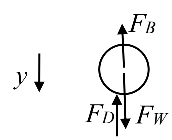
Thus, the flow speed for this case is $\boxed{U_\infty = 82.7\text{ m/s}}$.

A buoyant ball of specific gravity, SG< 1, dropped into water at an impact speed, V_0, penetrates a distance, h, into the water and pops out again. Assuming a constant drag coefficient, derive an expression for h as a function of the system properties. How deep will a 5 cm diameter ball with SG=0.5 and C_D= 0.47 penetrate if it enters water at a speed of 10 m/s? You may neglect splashing, air entrainment, and added mass effects in your analysis.



SOLUTION:

Apply Newton's 2nd Law to the ball:



$$m\frac{dV}{dt} = F_W - F_B - F_D \tag{1}$$

where m is the ball mass, y is the depth of the ball from the free surface, F_W is the ball weight, F_B is the buoyant force acting on the ball, and F_D is the drag force acting on the ball.

$$m = \rho_S \tfrac{\pi}{6} d^3 \tag{2}$$

$$F_W = mg \tag{3}$$

$$F_B = \rho_F \tfrac{\pi}{6} d^3 g \tag{4}$$

$$F_D = C_D \tfrac{1}{2} \rho_F V^2 \tfrac{\pi}{4} d^2 \tag{5}$$

where ρ_S and ρ_F are the ball and fluid densities, respectively, d is the ball diameter, g is the acceleration due to gravity, and C_D is the drag coefficient. Substitute Eqs. (2) - (5) into Eq. (1) and simplify.

$$\rho_S \tfrac{\pi}{6} d^3 \frac{dV}{dt} = \rho_S \tfrac{\pi}{6} d^3 g - \rho_F \tfrac{\pi}{6} d^3 g - C_D \tfrac{1}{2} \rho_F V^2 \tfrac{\pi}{4} d^2 \tag{6}$$

$$\frac{dV}{dt} = \left(1 - \frac{\rho_F}{\rho_S}\right) g - \frac{3}{4} C_D \frac{\rho_F}{\rho_S} \frac{1}{d} V^2 \tag{7}$$

$$\frac{dV}{dt} = \underbrace{\left(1 - \frac{1}{SG}\right) g}_{=-\alpha} - \underbrace{\frac{3}{4} C_D \frac{1}{SG} \frac{1}{d}}_{=\beta} V^2 \tag{8}$$

$$\frac{dV}{dt} = -\left(\alpha + \beta V^2\right) \tag{9}$$

where

$$\alpha = \left(\frac{1}{SG} - 1\right) g \tag{10}$$

$$\beta = \frac{3}{4} C_D \frac{1}{SG} \frac{1}{d} \tag{11}$$

Make Eq. (9) dimensionless using a dimensionless velocity and time:

$$V' = \sqrt{\frac{\beta}{\alpha}}V \tag{12}$$

$$t' = \sqrt{\alpha\beta}t \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (9) gives:

$$\frac{d\left(\sqrt{\frac{\alpha}{\beta}}V'\right)}{d\left(t'/\sqrt{\alpha\beta}\right)} = -\left[\alpha + \beta\left(\sqrt{\frac{\alpha}{\beta}}V'\right)^2\right] \tag{14}$$

$$\frac{dV'}{dt'} = -\left(1+V'^2\right) \tag{15}$$

The initial condition for Eq. (9) is:

$$V'\left(t'=0\right) = V_0' \tag{16}$$

where

$$V_0' = \sqrt{\frac{\beta}{\alpha}}V_0 \tag{17}$$

Solving Eq. (9) using an integration table or a symbolic ODE solver (e.g., MAPLE) gives:

$$\int_{V'=V_0'}^{V'=V'} \frac{dV'}{1-V'^2} = \int_{t'=0}^{t'=t'} dt' \tag{18}$$

$$-\tan^{-1}\left(V'\right) + \tan^{-1}\left(V_0'\right) = t' \tag{19}$$

$$V' = \tan\left[\tan^{-1}\left(V_0'\right) - t'\right] \tag{20}$$

Note that the maximum depth of the ball, h, occurs when $V'(t' = T') = 0$.

$$T' = \tan^{-1}\left(V_0'\right) \tag{21}$$

The maximum dimensionless depth of the ball, h' ($= \beta h$) is found by integrating Eq. (19) in time.

$$\int_{y'=0}^{y'=h'} dy' = \int_{t'=0}^{t'=T'} \tan\left[\tan^{-1}\left(V_0'\right) - t'\right]dt' \tag{22}$$

$$h' = -\frac{1}{2}\ln\left(1+\left\{\tan\left[\tan^{-1}\left(V_0'\right)+T'\right]\right\}^2\right) + \frac{1}{2}\ln\left(1+V_0'^2\right) \tag{23}$$

$$\boxed{\therefore h' = \frac{1}{2}\ln\left(\frac{1+V_0'^2}{1+\left\{\tan\left[2\tan^{-1}\left(V_0'\right)\right]\right\}^2}\right)} \tag{24}$$

A plot of the dimensionless velocity and position as functions of dimensionless time are shown in Fig. 1 using the data given in the problem statement.

SG = 0.5
d = 0.05 m
g = 9.81 m/s^2
C_D = 0.47
V_0 = 10 m/s
$\Rightarrow$ $\alpha = 9.81$ m/s^2 and $\beta = 14.1$ m^{-1} and $V_0' = 11.99$

Using the given data, the time at which the ball achieves its maximum depth is:

$$T' = 1.49 \Rightarrow \boxed{T = 0.13 \text{ s}} \tag{25}$$

The maximum depth is:

$$h' = 2.47 \Rightarrow \boxed{h = 0.18 \text{ m}} \tag{26}$$

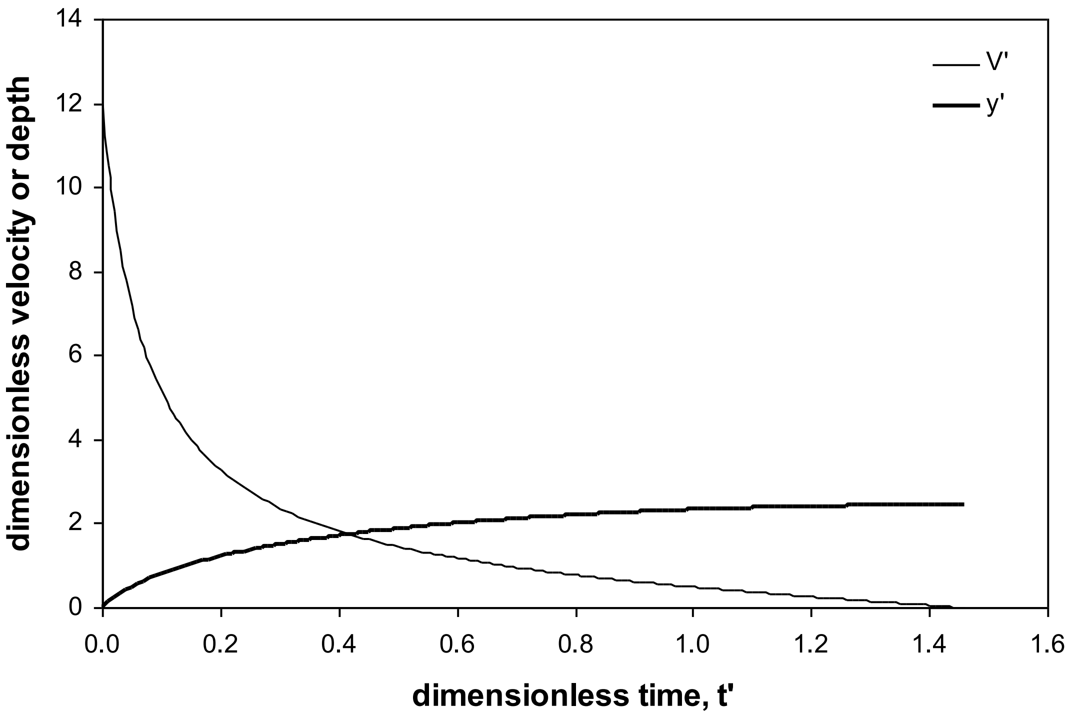
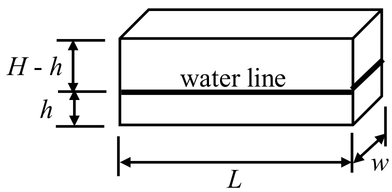


Figure 1. The dimensionless velocity, V', and dimensionless position, y', plotted as a function of dimensionless time, t', for $\alpha = 9.81$ m/s^2, $\beta = 14.1$ m^{-1}, and $V_0' = 11.99$.

In this analysis a constant drag coefficient was assumed. This is a reasonable assumption over the range $1000 < Re_d < 200{,}000$. A more accurate analysis would take into account the variation in drag coefficient with speed (and would also require a computational solution). In addition to splashing and air entrainment effects (air entrained into the wake of the ball), added mass effects should also be taken into account. When accelerating (or decelerating) an object in a fluid, we must also accelerate (or decelerate) the surrounding fluid. This extra force required to accelerate the surrounding fluid can be incorporated into the object mass and is known as an "added mass" or "virtual mass."

A barge weighing 8820 kN that is 10 m wide, 30 m long, and 7 m tall has come free from its tug boat in the Mississippi River. It is in a section of river that has a current of 1 m/s. In addition, there is a wind blowing straight upriver at 10 m/s . Assume that the drag coefficient is 1.3 for both the part of the barge in the wind as well as the part below the water. The drag coefficients for the water-exposed and air-exposed portions of the barge are based on the water and air wetted areas, respectively. Determine the speed at which the barge will be steadily moving. Is it moving upriver or downriver?

SOLUTION:



First determine the wetted areas above and below the waterline. Balancing forces in the vertical direction on the barge,

$$\sum F_{\text{vert}} = 0 = -W + \rho_{H_2O} g L w h \, , \tag{1}$$

where W is the barge weight and the second term on the right hand side is the buoyant force, with h being the draft of the barge (the depth below the water). Solving for h gives,

$$h = \frac{W}{\rho_{H_2O} g L w} \, . \tag{2}$$

Using the given values,

W = 8820 kN,
ρ_{H2O} = 1000 kg/m^3,
g = 9.81 m/s^2,
L = 30 m,
w = 10 m,
=> $\underline{h = 3.00 \text{ m.}}$

Now determine the wetted areas below and above water,

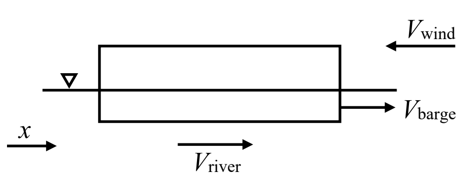
$$A_{\substack{\text{wetted,}\\\text{below}}} = Lw + 2Lh + 2wh \, , \tag{3}$$

$$A_{\substack{\text{wetted,}\\\text{above}}} = Lw + 2L(H-h) + 2w(H-h) \, . \tag{4}$$

Using the given values,

$\underline{A_{\text{wetted,below}} = 540 \text{ m}^2}$,
$\underline{A_{\text{wetted,above}} = 620 \text{ m}^2}$.

Now balance forces in the horizontal direction. These forces include the drag caused by the river and the drag caused by the wind. Assume that the barge is moving in the same direction as the river (downstream), as shown in the figure below.



$$\sum F_x = 0 = c_{D,\text{below}} \tfrac{1}{2}\rho_{H_2O}\left(V_{\text{river}} - V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{below}}} - c_{D,\text{above}} \tfrac{1}{2}\rho_{\text{air}}\left(V_{\text{wind}} + V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{above}}}, \tag{5}$$

Solve for V_{barge}, noting that the drag coefficients are the same above and below the waterline (given in the problem statement),

$$\rho_{H_2O}\left(V_{\text{river}} - V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{below}}} = \rho_{\text{air}}\left(V_{\text{wind}} + V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{above}}}, \tag{6}$$

$$V_{\text{river}}^2 - 2V_{\text{river}}V_{\text{barge}} + V_{\text{barge}}^2 = \underbrace{\left(\frac{\rho_{\text{air}}}{\rho_{H_2O}}\right)\left(\frac{A_{\substack{\text{wetted,}\\ \text{above}}}}{A_{\substack{\text{wetted,}\\ \text{below}}}}\right)}_{=c}\left(V_{\text{wind}}^2 + 2V_{\text{wind}}V_{\text{barge}} + V_{\text{barge}}^2\right), \tag{7}$$

$$\left(1-c\right)V_{\text{barge}}^2 - 2\left(V_{\text{river}} + cV_{\text{wind}}\right)V_{\text{barge}} + V_{\text{river}}^2 - cV_{\text{wind}}^2 = 0, \tag{8}$$

$$\underbrace{1}_{=A} V_{\text{barge}}^2 + \underbrace{\frac{-2\left(V_{\text{river}} + cV_{\text{wind}}\right)}{\left(1-c\right)}}_{=B} V_{\text{barge}} + \underbrace{\frac{\left(V_{\text{river}}^2 - cV_{\text{wind}}^2\right)}{\left(1-c\right)}}_{=C} = 0, \tag{9}$$

$$V_{\text{barge}} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \tag{10}$$

Using the give data,

$\rho_{\text{air}} = 1.23\ \text{kg/m}^3$,
$V_{\text{river}} = 1$ m/s,
$V_{\text{wind}} = 10$ m/s,
=> $c = 1.41 \times 10^{-3}$, $A = 1$, $B = -2.03$ m/s, $C = 8.60 \times 10^{-1}\ \text{m}^2/\text{s}^2$
=> $\underline{V_{\text{barge}} = 1.43\ \text{m/s},\ 0.601\ \text{m/s}.}$

Note that it's not possible for the barge to move faster than the river's speed of 1 m/s, so $V_{\text{barge}} \neq 1.43$ m/s. Thus, the correct answer is $\boxed{V_{\text{barge}} = 0.601\ \text{m/s (downstream)}}$.

If we had assumed that V_{barge} was moving upstream (same direction as V_{wind}), then Eq. (5) would be,

$$\sum F_x = 0 = c_{D,\text{below}} \tfrac{1}{2}\rho_{H_2O}\left(V_{\text{river}} + V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{below}}} - c_{D,\text{above}} \tfrac{1}{2}\rho_{\text{air}}\left(V_{\text{wind}} - V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{above}}}, \tag{11}$$

which would simplify to,

$$V_{\text{barge}}^2 + \frac{2\left(V_{\text{river}} + cV_{\text{wind}}\right)}{\left(1-c\right)} V_{\text{barge}} + \frac{\left(V_{\text{river}}^2 - cV_{\text{wind}}^2\right)}{\left(1-c\right)} = 0. \tag{12}$$

Solving this equation gives,

$\underline{V_{\text{barge}} = -0.601\ \text{m/s},\ -1.43\ \text{m/s}}$.

Thus, we see that the original choice of direction for V_{barge} (upstream) was incorrect and the barge is actually moving downstream. As in the previous discussion, the barge cannot move faster than the river speed so the correct speed is 0.601 m/s.

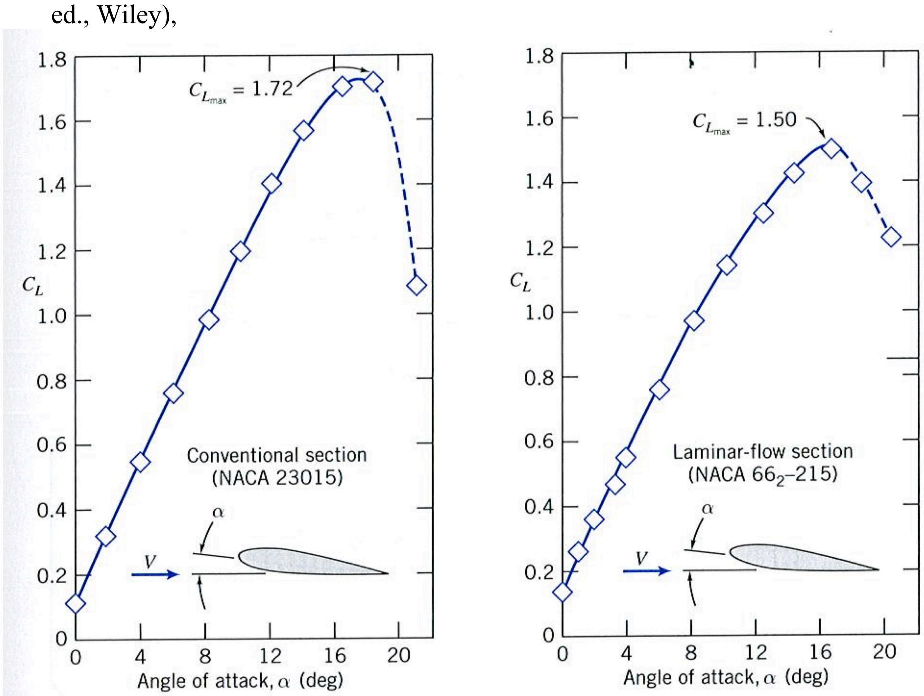
Some cars come with a rear "spoiler" (actually an upside-down airfoil) mounted on the rear of the vehicle that is supposed to increase the down force on the car and improve traction. Calculate a typical down force caused by a rear wing used on a passenger vehicle.

SOLUTION:
The lift force is given by,

$$L = C_L \tfrac{1}{2}\rho V^2 A \,, \tag{1}$$

where,

A = 2 ft^2 (= 0.186 m^2), assuming a span of 4 ft and a chord length of 0.5 ft (note that this is a planform area),
ρ = 1.23 kg/m^3,
V = 24.6 m/s (= 55 mph),
C_L = 1.4, (a typical value based on Fig. 9.17 from Pritchard et al., *Introduction to Fluid Mechanics*, 8th ed., Wiley),



(a) Lift coefficient vs. angle of attack

=> $\boxed{L = 96.9 \text{ N } (= 21.8 \text{ lb}_f)}$

Thus, we see the spoiler produces very little down force on the vehicle.

To produce a down force of 200 lb$_f$ (= 890 N), the car would need to travel at a speed of 70.7 m/s (= 158 mph).

Note that rear spoilers are sometimes used to direct airflow downward to help reduce the size of the trailing wake and thus reduce drag.

9.11. Review Questions

(1) What scaling arguments are used in deriving the boundary layer equations?
(2) What are the appropriate boundary conditions for the boundary layer equations?
(3) What restrictions are there on the Reynolds number for using the boundary layer equations?
(4) Describe how the pressure within a boundary layer is determined.
(5) Describe, in words, the approach used in deriving the Blasius solution to the boundary layer equations.
(6) What assumptions are made in the Blasius boundary layer solution? (e.g., Reynolds number limitations, pressure gradients, free stream conditions, surface curvature, etc.)
(7) At what Reynolds number (an engineering rule of thumb estimate) does a laminar boundary layer transition to a turbulent boundary layer?
(8) How does the boundary layer thickness vary with the distance from the leading edge of the boundary layer for a flat plate, no pressure gradient boundary layer flow?
(9) What is the expression for the 99% boundary layer thickness resulting from the Blasius solution?
(10) What do the Falkner-Skan boundary layer solutions represent?
(11) What are the boundary conditions used in the Falkner-Skan boundary layer solution?
(12) Give two examples of practical boundary layer solutions that are embedded within the Falkner-Skan general solution.
(13) Can the Kärmän momentum integral equation (KMIE) be used for flows with non-uniform pressure gradients? Turbulent flows? Compressible flows? Unsteady flows?
(14) How might one find the outer flow velocity, U, when using the KMIE?
(15) Describe the typical methodology used when applying the KMIE.
(16) What is the 1/7th power law profile for a turbulent boundary layer?
(17) In which type of boundary layer flow does the shear stress decrease most rapidly? Laminar or turbulent? In which type of flow does the drag increase most rapidly?
(18) Give a physical description of why boundary layer separation occurs.
(19) What defines the point at which boundary layer separation occurs?
(20) Why can't the boundary layer equations be used downstream of boundary layer separation point?
(21) Why do turbulent boundary layers separate further downstream than laminar boundary layers?
(22) What is meant by "favorable" and "adverse" pressure gradients?
(23) Can a boundary layer separate in a favorable pressure gradient flow?
(24) Must boundary layers always separate in an adverse pressure gradient flow?
(25) What are the restrictions in using Thwaites' correlation?
(26) Describe the flow behavior as a function of Reynolds number for flow over a cylinder.
(27) Sketch a plot of drag coefficient as a function of Reynolds number for flow over a sphere. Indicate points of particular interest on the plot. Identify whether the axes are linear or logarithmic.

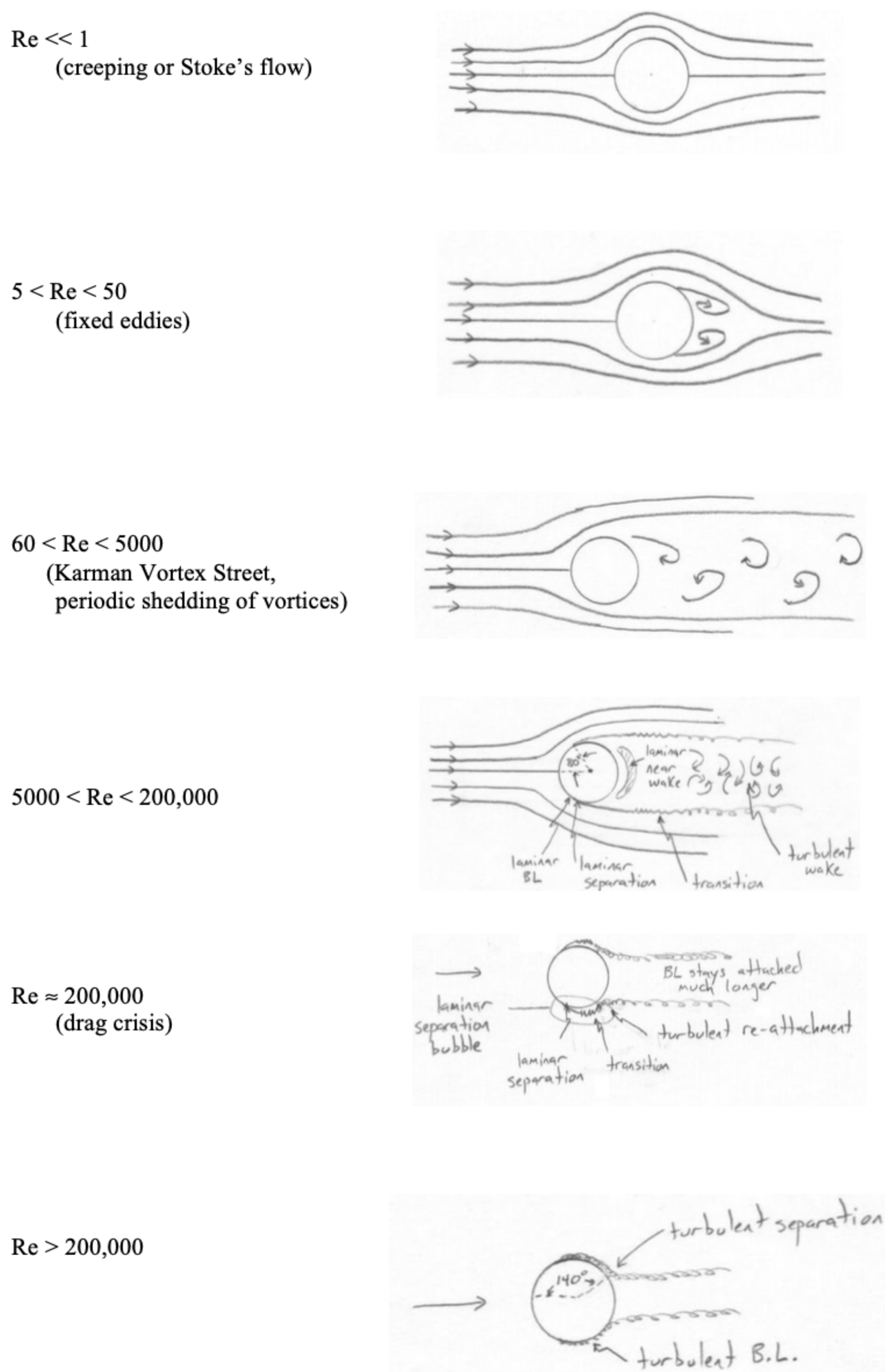


FIGURE 9.29. Drawings of the different regimes of flow around a sphere as a function of Reynolds number.

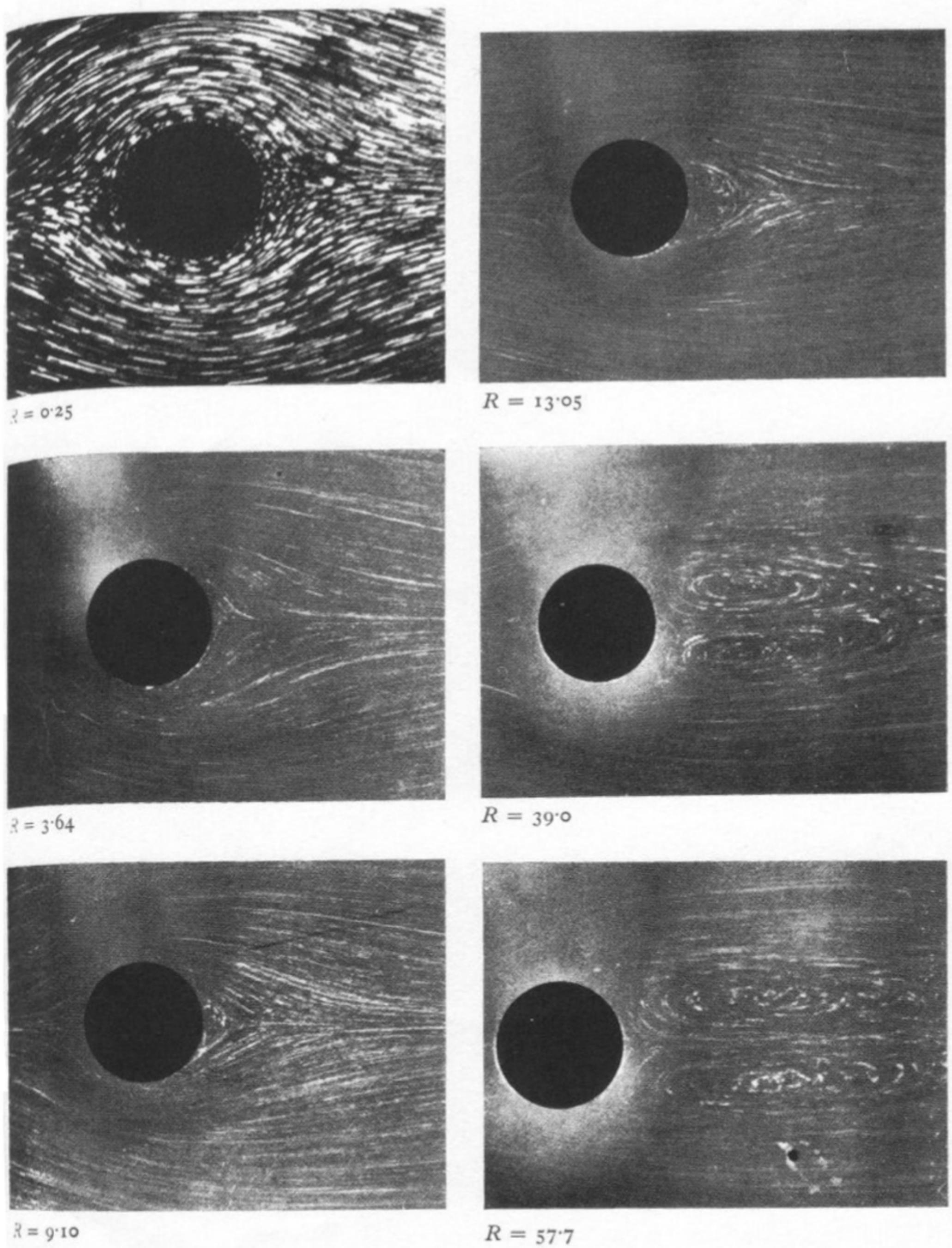
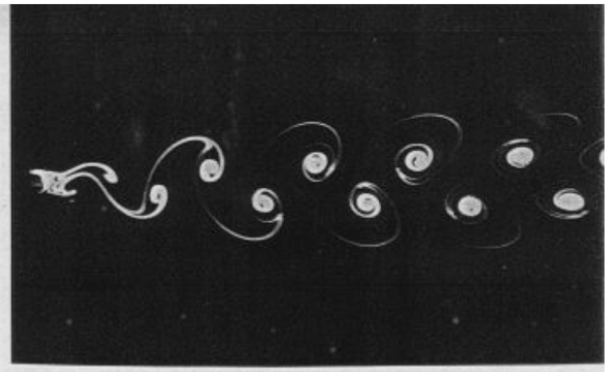


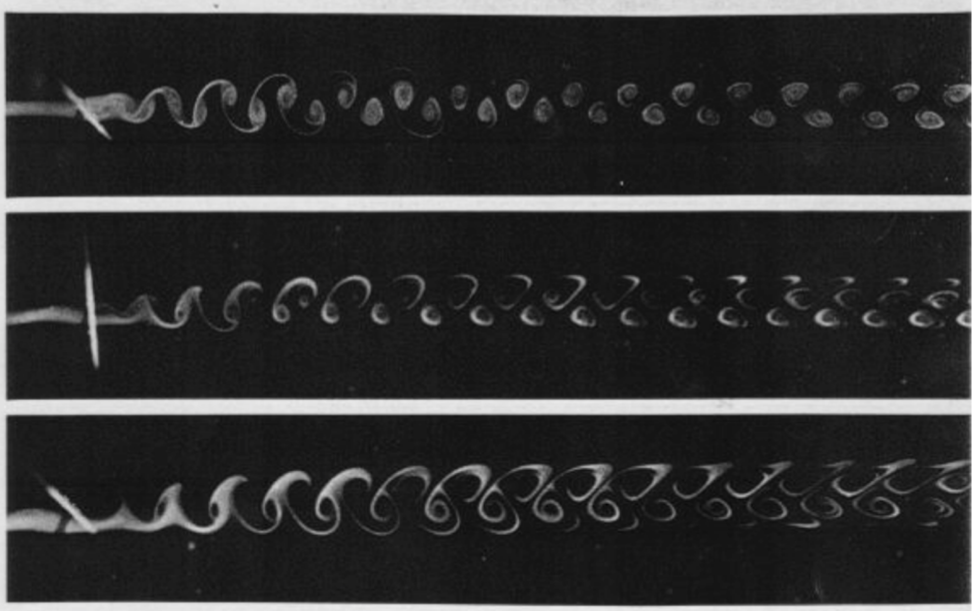
Figure 4.12.1. Streamlines of steady flow (from left to right) past a circular cylinder of radius a; $R = 2aU/\nu$. The photograph at $R = 0{\cdot}25$ (from Prandtl and Tietjens 1934) shows the movement of solid particles at a free surface, and all the others (from Taneda 1956*a*) show particles illuminated over an interior plane normal to the cylinder axis.

(From Batchelor, G.K., *An Introduction to Fluid Dynamics*, Cambridge University Press.)

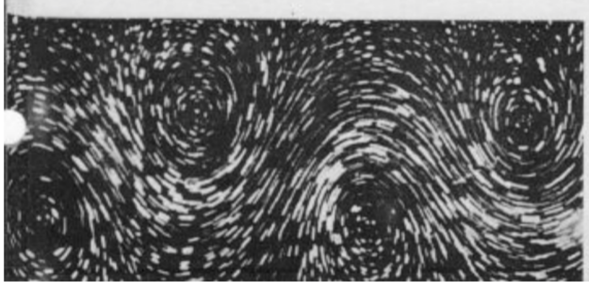
Figure 9.30. Photographs showing the different regimes of flow around a sphere as a function of Reynolds number (R). These photographs are from Batchelor, G.K., *An Introduction to Fluid Dynamics*, Cambridge University Press.

96. Kármán vortex street behind a circular cylinder at R=105. The initially spreading wake shown opposite develops into the two parallel rows of staggered vortices that von Kármán's inviscid theory shows to be stable when the ratio of width to streamwise spacing is 0.28. Streaklines are shown by electrolytic precipitation in water. *Photograph by Sadatoshi Taneda*





97. Smoke at various levels in a vortex street. A smoke filament in air shows, at a Reynolds number of 100, both shear layers (top photographs), only one shear layer (middle), and the irrotational flow below the wake (bottom). *Zdravkovich 1969*



98. Kármán vortices in absolute motion. Here the camera moves with the vortices rather than the cylinder. The streamline pattern closely resembles the inviscid one calculated by von Kármán. The flow is visualized by particles floating on water. *Photograph by R. Wille, from Werlé 1973. Reproduced, with permission, from the* Annual Review of Fluid Mechanics, *Volume 5, © 1973 by Annual Reviews Inc.*

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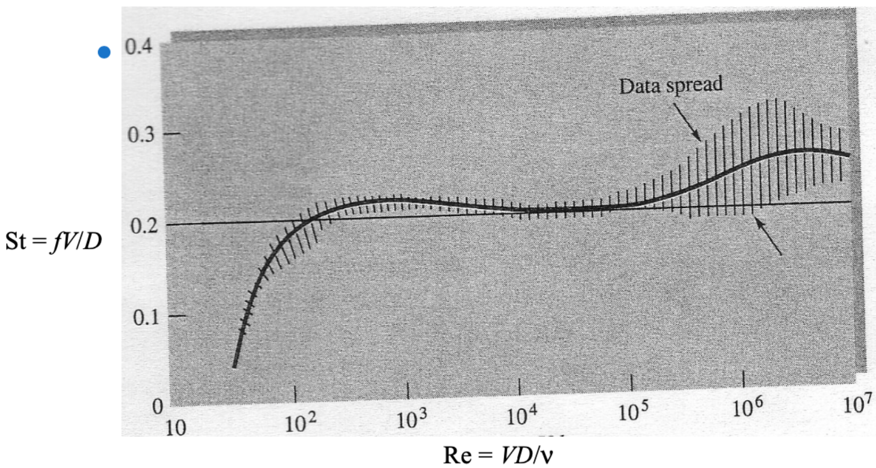
(From Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press.)

Figure 9.31. Photographs showing Kármán vortex streets downstream of an immersed object. These photos are from Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press.

FIGURE 9.32. Photographs of the Tacoma Narrows Bridge prior to failure.



(Figure from White, F.M., *Fluid Mechanics*, McGraw-Hill.)

FIGURE 9.33. The Strouhal number based on the vortex shedding frequency from a cylinder plotted as a function of the Reynolds number. This figure is from White, F.M., *Fluid Mechanics*, McGraw-Hill.

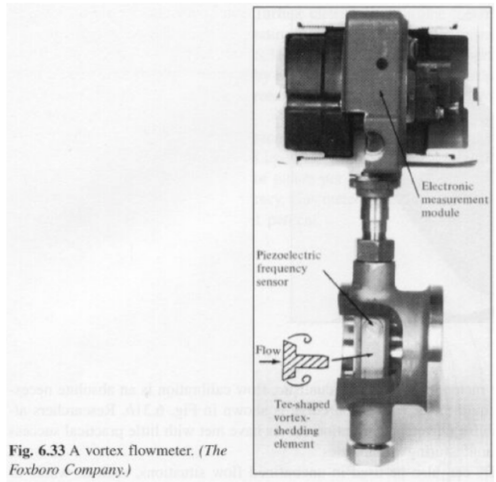


Figure 9.34. A photograph of a vortex flow meter.

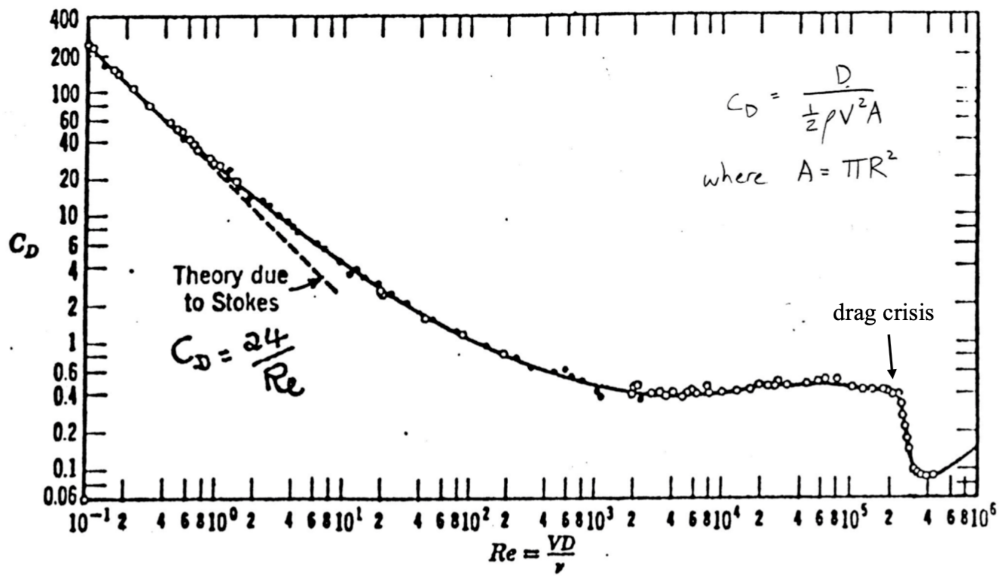


Fig. 8.32 Drag coefficient of a sphere as a function of Reynolds number (Ref. 13).

Figure 9.35. The drag coefficient for a sphere based on the sphere's frontal projected area plotted against the Reynolds number.

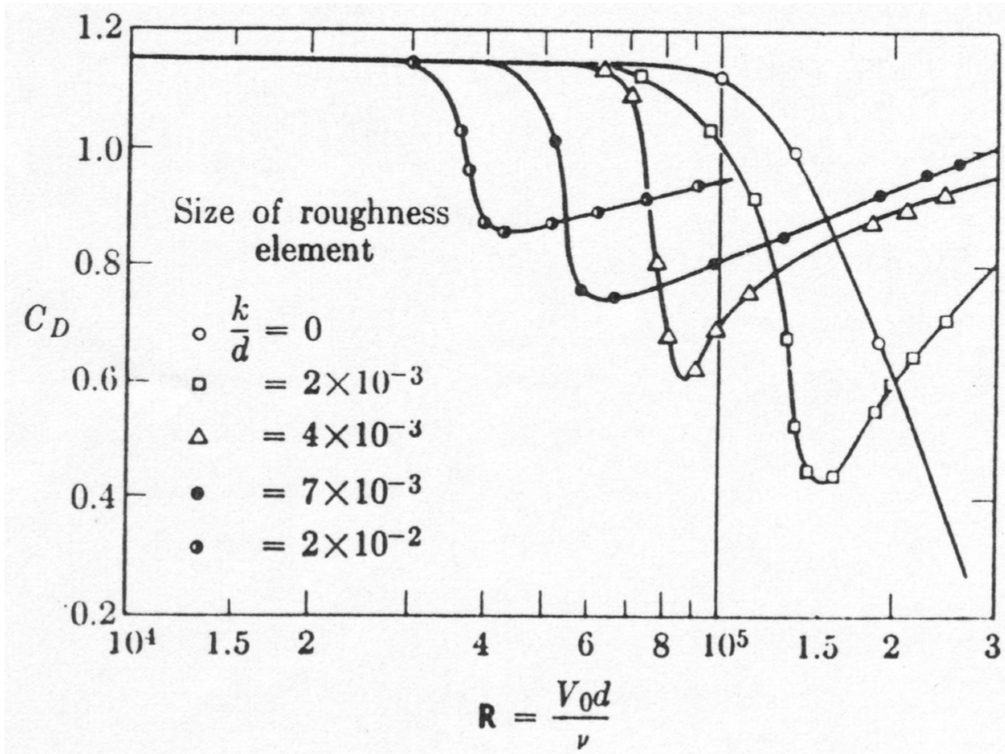


Figure 9.36. The drag coefficient on a cylinder plotted as a function of the Reynolds number for different degrees of surface roughness. Increasing roughness causes the drag crisis to occur at a smaller Reynolds number.

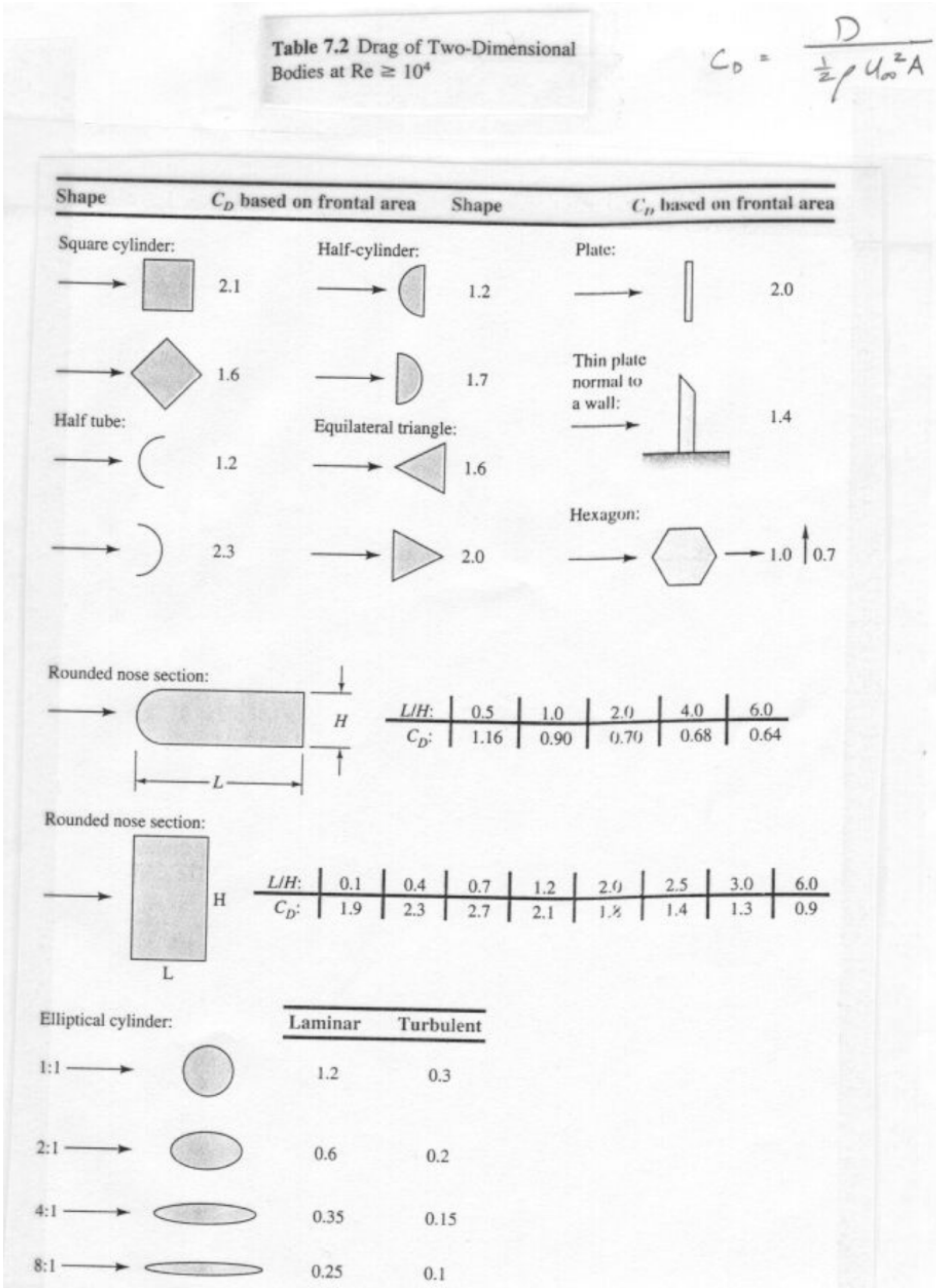


Table 7.2 Drag of Two-Dimensional Bodies at Re ≥ 10^4

$C_D = \frac{D}{\frac{1}{2}\rho U_\infty^2 A}$

Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder:	2.1	Half-cylinder:	1.2	Plate:	2.0
	1.6		1.7	Thin plate normal to a wall:	1.4
Half tube:	1.2	Equilateral triangle:	1.6		
	2.3		2.0	Hexagon:	1.0, 0.7

Rounded nose section:

L/H:	0.5	1.0	2.0	4.0	6.0
C_D:	1.16	0.90	0.70	0.68	0.64

Rounded nose section:

L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9

Elliptical cylinder:

	Laminar	Turbulent
1:1	1.2	0.3
2:1	0.6	0.2
4:1	0.35	0.15
8:1	0.25	0.1

Figure 9.37. Drag coefficients for a variety of two-dimensional objects. This table is from White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.

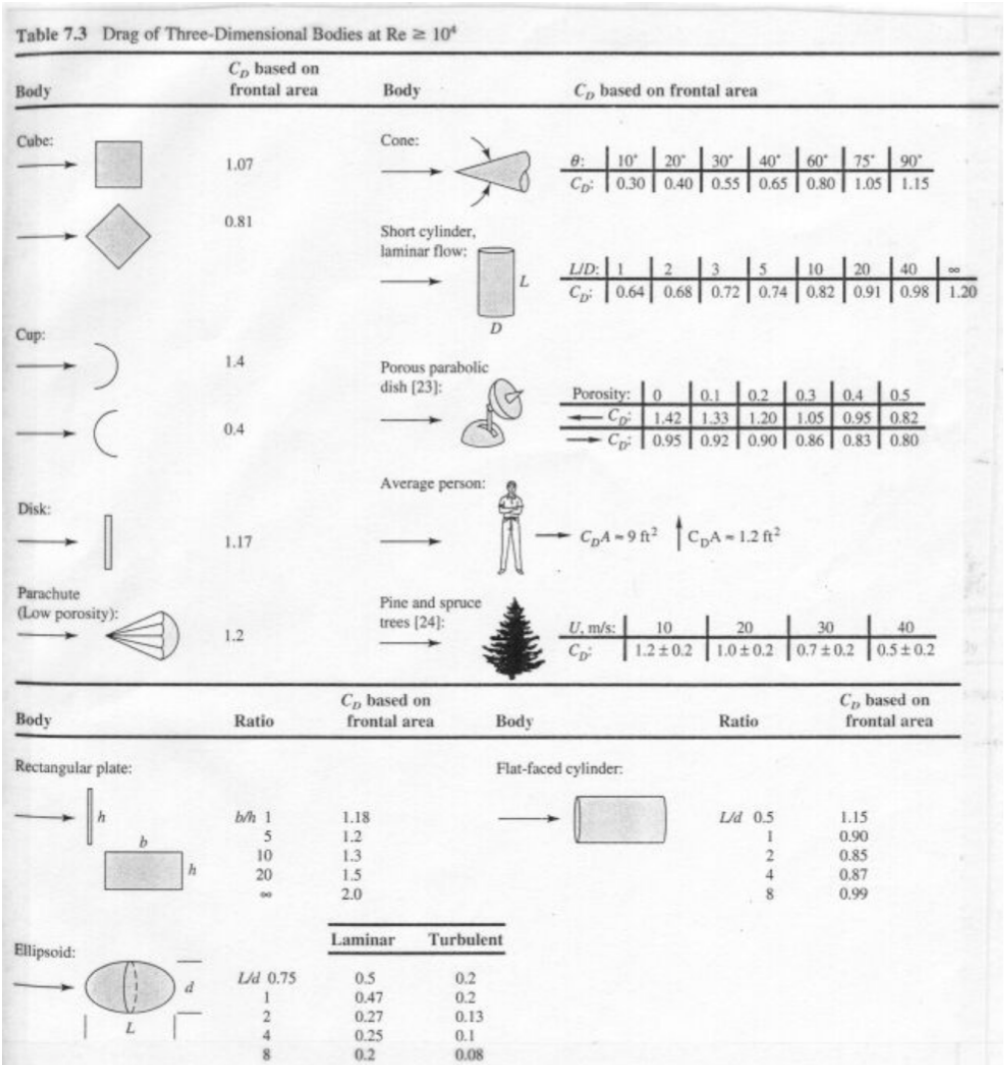


Table 7.3 Drag of Three-Dimensional Bodies at Re $\geq 10^4$

Body	C_D based on frontal area
Cube:	1.07
Cube (rotated 45°)	0.81
Cup:	1.4
Cup (reversed)	0.4
Disk:	1.17
Parachute (Low porosity):	1.2

Body	C_D based on frontal area							
Cone:	θ:	10°	20°	30°	40°	60°	75°	90°
	C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15
Short cylinder, laminar flow:	L/D:	1	2	3	5	10	20	40	∞
	C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

Body							
Porous parabolic dish [23]:	Porosity:	0	0.1	0.2	0.3	0.4	0.5
	← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
	→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

Average person: → $C_DA \approx 9\ \text{ft}^2$ ↑ $C_DA \approx 1.2\ \text{ft}^2$

Body					
Pine and spruce trees [24]:	U, m/s:	10	20	30	40
	C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

Body	Ratio	C_D based on frontal area
Rectangular plate:	b/h 1	1.18
	5	1.2
	10	1.3
	20	1.5
	∞	2.0

Body	Ratio	C_D based on frontal area
Flat-faced cylinder:	L/d 0.5	1.15
	1	0.90
	2	0.85
	4	0.87
	8	0.99

Body	Ratio	Laminar	Turbulent
Ellipsoid:	L/d 0.75	0.5	0.2
	1	0.47	0.2
	2	0.27	0.13
	4	0.25	0.1
	8	0.2	0.08

Figure 9.38. Drag coefficients for a variety of three-dimensional objects. This table is from White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.