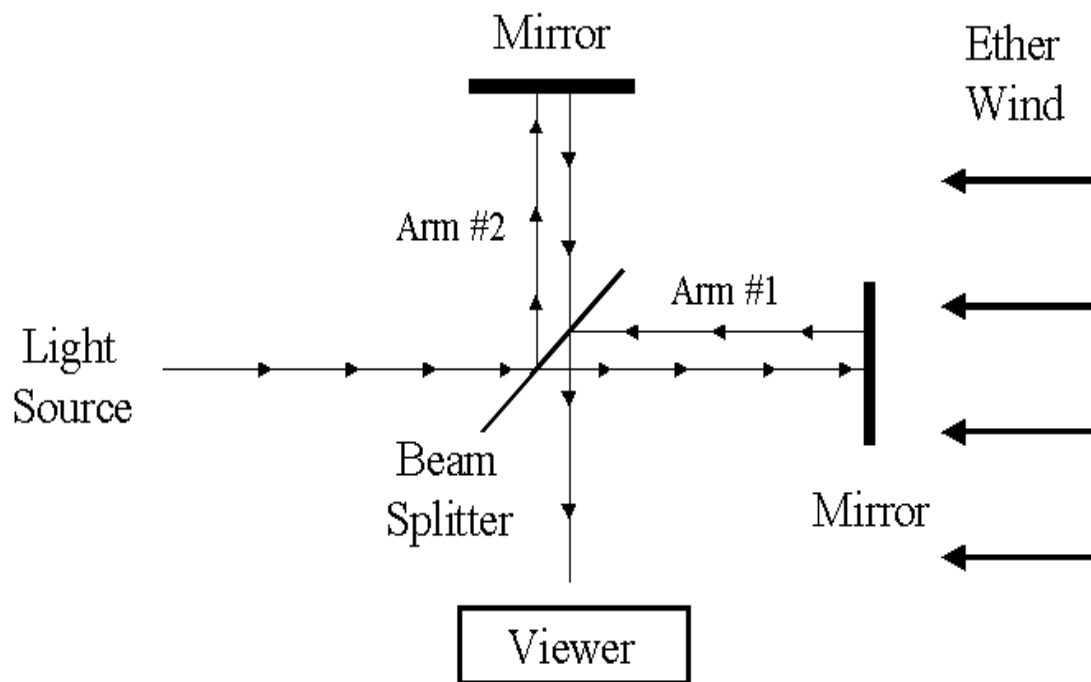


2.2 The Michelson-Morley Experiment

Note. Sound waves need a medium through which to travel. In 1864 James Clerk Maxwell showed that light is an electromagnetic wave. Therefore it was assumed that there is an *ether* which propagates light waves. This ether was assumed to be everywhere and unaffected by matter. This ether could be used to determine an absolute reference frame (with the help of observing how light propagates through the ether).

Note. The Michelson-Morley experiment (circa 1885) was performed to detect the Earth's motion through the ether as follows:



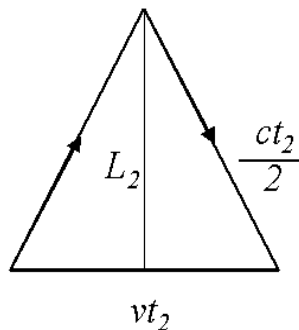
The viewer will see the two beams of light which have traveled along different

arms display some interference pattern. If the system is rotated, then the influence of the “ether wind” *should* change the time the beams of light take to travel along the arms and therefore should change the interference pattern. The experiment was performed at different times of the day and of the year. **NO CHANGE IN THE INTERFERENCE PATTERN WAS OBSERVED!**

Example (Exercise 2.2.2). Suppose L_1 is the length of arm #1 and L_2 is the length of arm#2. The speed of a photon (relative to the source) on the trip “over” to the mirror is $c - v$ and so takes a time of $L_1/(c - v)$. On the return trip, the photon has speed of $c + v$ and so takes a time of $L_1/(c + v)$. Therefore the round trip time is

$$\begin{aligned} t_1 &= \frac{L_1}{c - v} + \frac{L_1}{c + v} = \frac{L_1(c + v) + L_1(c - v)}{c^2 - v^2} = \frac{2cL_1}{c^2 - v^2} \\ &= \frac{2L_1}{c} \frac{1}{1 - v^2/c^2} = \frac{2L_1}{c} \left(1 - \frac{v^2}{c^2}\right)^{-1}. \end{aligned}$$

The photon traveling along arm #2 must follow a path (relative to a “stationary” observer) of



We need the time the photon travels (t_2) and the angle at which the photon

leaves the mirror to be such that

$$L_2^2 + \left(\frac{vt_2}{2}\right)^2 = \left(\frac{ct_2}{2}\right)^2$$

(the photon must travel with a component of velocity “upstream” to compensate for the wind). Then

$$\begin{aligned} L_2^2 &= \left(\frac{ct_2}{2}\right)^2 - \left(\frac{vt_2}{2}\right)^2 \\ L_2^2 &= t_2^2 \left(\frac{c^2}{4} - \frac{v^2}{4}\right) \\ t_2 &= \frac{2L_2}{\sqrt{c^2 - v^2}} = \frac{2L_2}{c} \frac{1}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

Now $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$, so

$$t_1 \approx \frac{2L_1}{c} \left(1 + \frac{v^2}{c^2}\right).$$

Also,

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \dots$$

and with $m = -1/2$ and $x = -v^2/c^2$,

$$t_2 \approx \frac{2L_2}{c} \left(1 + \left(\frac{-1}{2}\right) \left(\frac{-v^2}{c^2}\right)\right) = \frac{2L_2}{c} \left(1 + \frac{v^2}{2c^2}\right).$$

For the Earth’s orbit around the sun, $v/c \approx 10^{-4}$ so the approximation is appropriate. Now the rays recombine at the viewer separated by

$$\Delta t = t_1 - t_2 \approx \frac{2}{c} \left(L_1 - L_2 + \frac{L_1 v^2}{c^2} - \frac{L_2 v^2}{2c^2}\right).$$

Now suppose the apparatus is rotated 90° so that arm #1 is now transverse to the ether wind. Let t'_1 and t'_2 denote the new round trip light travel times. Then (as

above, replacing L_1 with L_2 in t_1 to determine t'_2 and replacing L_2 with L_1 in t_2 to determine t'_1):

$$t'_1 \approx \frac{2L_1}{c} \left(1 + \frac{v^2}{2c^2}\right) \quad \text{and} \quad t'_2 \approx \frac{2L_2}{c} \left(1 + \frac{v^2}{2c^2}\right).$$

Then

$$\Delta t' = t'_1 - t'_2 \approx \frac{2}{c}(L_1 - L_2) + \frac{v^2}{c^3}(L_1 - 2L_2)$$

and

$$\begin{aligned} \Delta t - \Delta t' &\approx \frac{2}{c}(L_1 - L_2) + \frac{2v^2}{c^3} \left(L_1 - \frac{L_2}{2}\right) - \left\{ \frac{2}{c}(L_1 - L_2) + \frac{v^2}{c^3}(L_1 - 2L_2) \right\} \\ &= \frac{v^2}{c^3}(L_1 + L_2). \end{aligned}$$

This is the time change produced by rotating the apparatus.

Note. In 1892, Fitzgerald proposed that an object moving through the ether wind with velocity v experiences a contraction in the direction of the ether wind of $\sqrt{1 - v^2/c^2}$. That is, in the diagram above, L_1 is contracted to $L_1\sqrt{1 - v^2/c^2}$ and then we get $t_1 = t_2$ when $L_1 = L_2$, potentially explaining the results of the Michelson-Morley experiment. This is called the Lorentz-Fitzgerald contraction. Even under this assumption, “it turns out” (see the following example) that the Michelson-Morley apparatus with unequal arms will exhibit a pattern shift over a 6 month period as the Earth changes direction in its orbit around the Sun. In 1932, Kennedy and Thorndike performed such an experiment and detected no such shift.

Example (Exercise 2.2.4). Suppose in the Michelson-Morley apparatus that $\Delta L = L_1 - L_2 \neq 0$ and that there is a contraction by a factor of $\sqrt{1 - v^2/c^2}$ in the direction of the ether wind. Then show

$$\Delta t = \frac{2}{c} \Delta L \left(1 + \frac{v^2}{2c^2} \right).$$

Solution. As in Exercise 2.2.1, we have

$$t_1 = \frac{2L_1}{c} \frac{1}{\sqrt{1 - v^2/c^2}} \quad t_2 = \frac{2L_2}{c} \frac{1}{\sqrt{1 - v^2/c^2}}$$

and so

$$\begin{aligned} \Delta t &= t_1 - t_2 = \frac{2L_1}{c} \frac{1}{\sqrt{1 - v^2/c^2}} - \frac{2L_2}{c} \frac{1}{\sqrt{1 - v^2/c^2}} \\ &= \frac{2}{c} (L_1 - L_2) \frac{1}{\sqrt{1 - v^2/c^2}} \\ &\approx \frac{2}{c} (L_1 - L_2) \left(1 + \left(\frac{-1}{2} \right) \left(\frac{-v^2}{c^2} \right) \right) \\ &= \frac{2}{c} (L_1 - L_2) \left(1 + \frac{v^2}{2c^2} \right). \end{aligned}$$

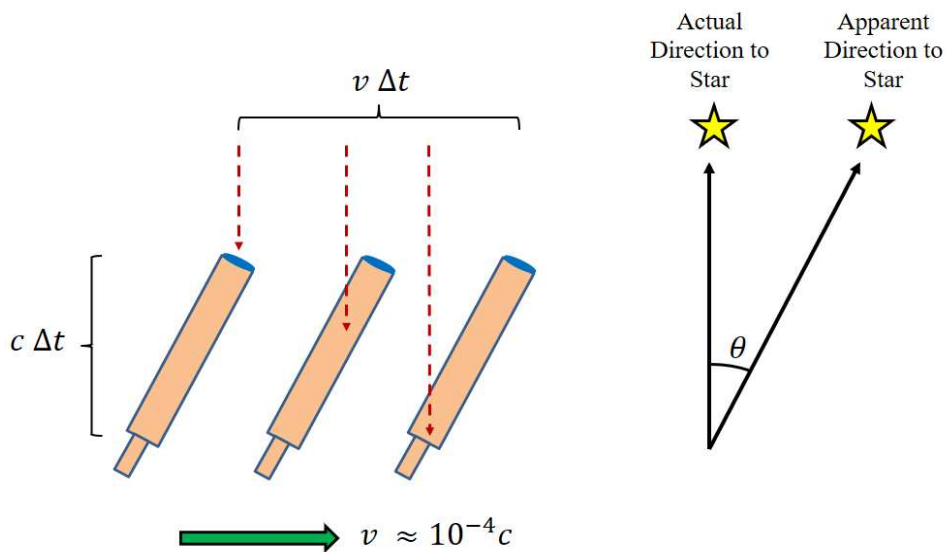
Note. Since the equation in the above exercise expresses Δt as a function of v only (c , L_1 , and L_2 being constant; although they may be contracted, but this is taken care of in the computations), we would see Δt vary with a period of 6 months (as mentioned above).

Note. Another suggestion to explain the negative result of the Michelson-Morley experiment was the idea that the Earth “drags the ether along with it” as it orbits the sun, galactic center, etc. This idea is rejected because of *stellar aberration*, discovered by James Bradley in 1725.

Example (Exercise 2.2.9). Light rays from a star directly overhead enter a telescope. Suppose the Earth, in its orbit around the Sun, is moving at a right angle to the incoming rays, in the direction indicated in the figure below. In the time it takes a ray to travel down the barrel to the eyepiece, the telescope will have moved slightly to the right. Therefore, in order to prevent the light rays from falling on the side of the barrel rather than on the eyepiece lens, we must tilt the telescope slightly from the vertical, if we are to see the star. Consequently, the *apparent* position of the star is displaced forward somewhat from the actual position. Show that the angle of displacement, θ , is (in radians)

$$\theta = \tan^{-1}(v/c) \approx v/c$$

where v is the Earth's orbital velocity. Verify that $\theta \approx 20.6''$ (roughly the angle subtended by an object 0.1 mm in diameter held at arm's length). This is also about 1% of the angular size of the moon in the night sky.



Solution. Clearly

$$\theta = \tan^{-1} \left(\frac{v\Delta t}{c\Delta t} \right) = \tan^{-1} \left(\frac{v}{c} \right).$$

Now

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \text{ for } |x| < 1$$

and so $\tan^{-1} \left(\frac{v}{c} \right) \approx \frac{v}{c}$. We have the Earth's orbital velocity implying $v/c = 10^{-4}$ (see Exercise 2.2.2). So

$$\theta = 10^{-4} = 10^{-4} \left(\frac{360^\circ}{2\pi} \right) \left(\frac{60'}{1^\circ} \right) \left(\frac{60''}{1'} \right) \approx 20.6''.$$

Note. As the Earth revolves around the Sun in its nearly circular annual orbit, the apparent position of the star will trace a circle with angular radius $20.6''$. This is indeed observed. If the Earth dragged a layer of ether along with it, the light rays, upon entering this layer, would acquire a horizontal velocity component matching the forward velocity v of the telescope. There would then be no aberration effect.

Conclusion. The speed of light is constant and the same in all directions and in all inertial frames.

Revised: 6/24/2019