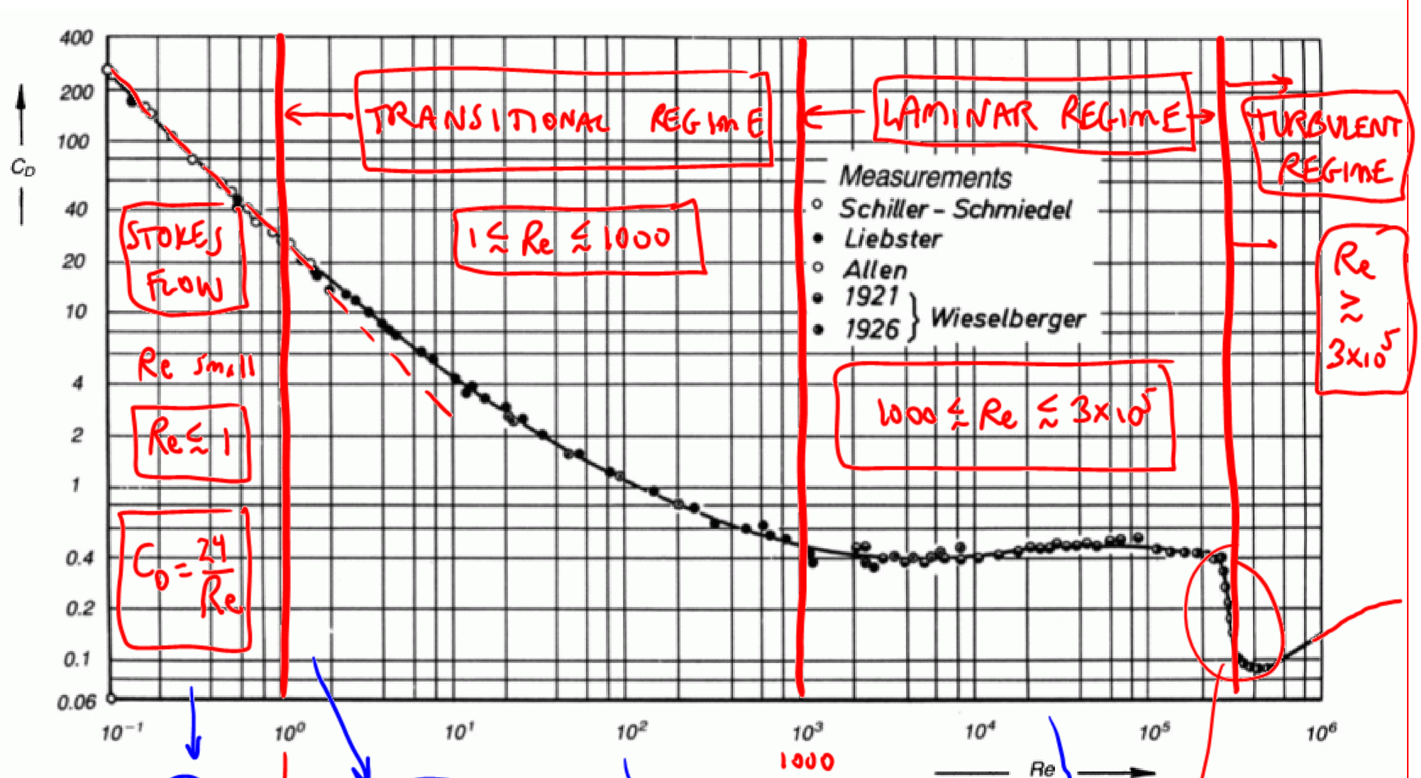
[AERODYNAMIC DRAG ON CYLINDERS AND SPHERES]()

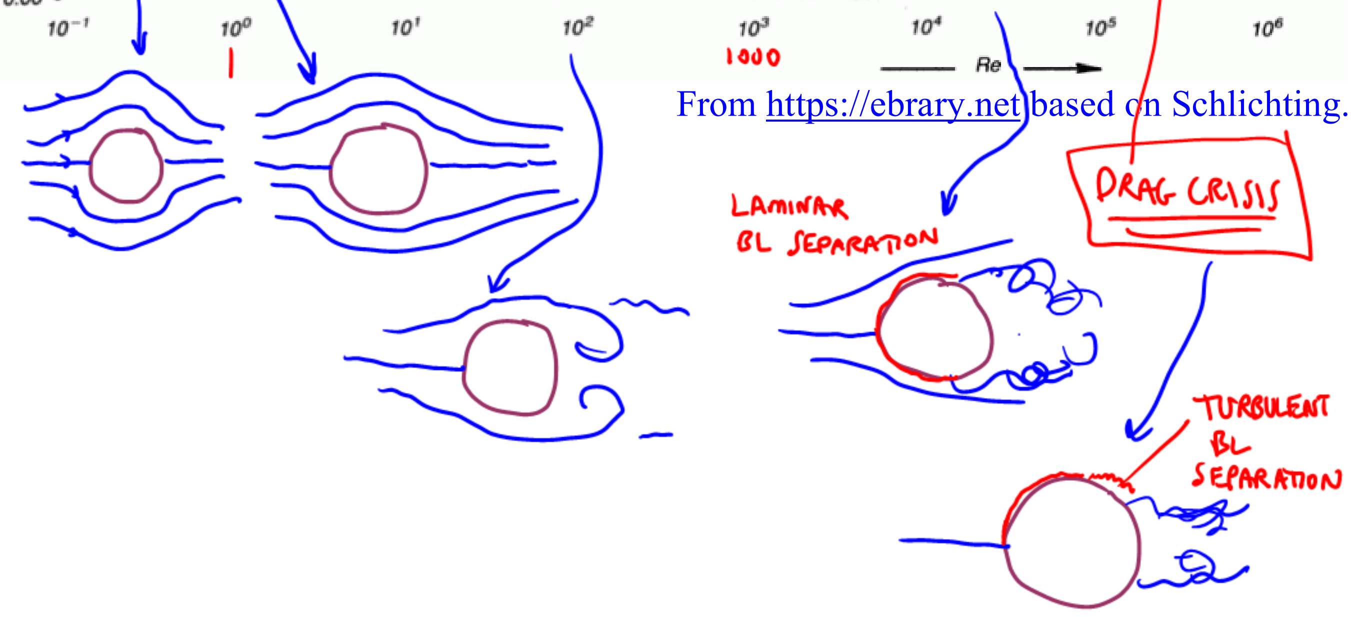
In this lesson, we will:

- Discuss how **Drag Coefficient** of **Spheres** and **Cylinders** varies with Reynolds number
- Show how to apply the **Morrison Equation** for sphere drag
- Define the **Drag Crisis** and how **Rough Walls** can sometimes lower drag (e.g., golf balls)
- Do some example problems

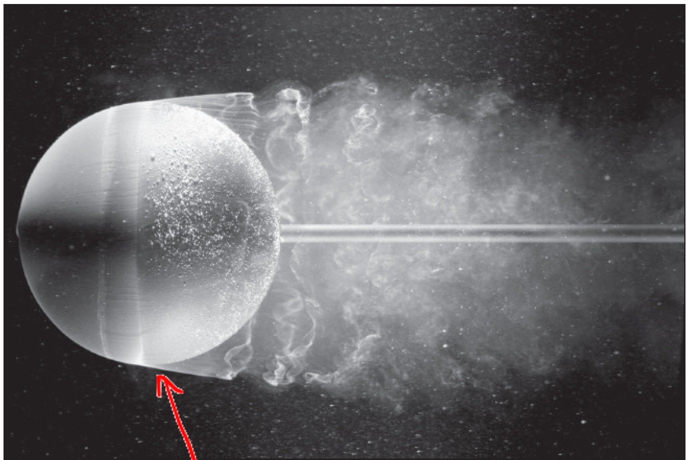
[Aerodynamic Drag on Smooth Spheres]()

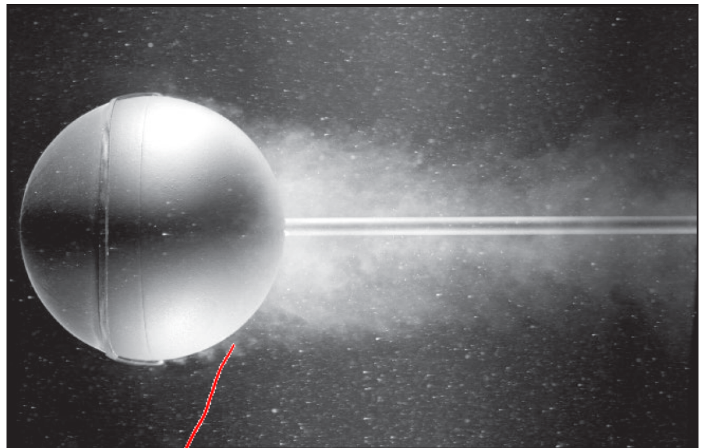
Experimental data show a huge range of C_D values for a sphere, depending on Reynolds number. This classic plot was first produced by Hermann Schlichting, *Boundary Layer Theory*, 1954.





From https://ebrary.net based on Schlichting.





From Çengel and Cimbala, Ed. 4.

LAMINAR BL SEPARATION

TURBULENT BL SEPARATION

In a 2016 paper, **Faith A. Morrison** created a curve fit equation for C_D of a sphere that spans the entire range of Reynolds number up to 10^6. Here is the **Morrison Equation**:

$$C_D \approx \frac{24}{\text{Re}} + \frac{2.6\left(\frac{\text{Re}}{5.0}\right)}{1+\left(\frac{\text{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\text{Re}}{10^6}\right)}{1+\left(\frac{\text{Re}}{10^6}\right)} \quad \text{for Re} < 10^6$$

Example: Drag coefficient on a sphere

Given: A 1.55 mm sphere is moving in air at a speed of 1.25 m/s. The air properties are:

- $\rho = 1.246\ \text{kg/m}^3$
- $\nu = 1.426\times10^{-5}\ \text{m}^2\text{/s}$

To do: Calculate the Reynolds number and the drag coefficient for this sphere.

Solution:

MORRISON EQ IS FOR SMOOTH SPHERES

$$\text{Re} = \frac{VD_p}{\nu}$$

$$C_D \approx \frac{24}{\text{Re}} + \frac{2.6\left(\frac{\text{Re}}{5.0}\right)}{1+\left(\frac{\text{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\text{Re}}{10^6}\right)}{1+\left(\frac{\text{Re}}{10^6}\right)} \quad \text{for Re} < 10^6$$

$$Re = \frac{(1.25\ \text{m/s})(1.55\ \text{mm})}{1.426\times10^{-5}\ \text{m}^2\text{/s}}\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right) \Rightarrow Re = 135.869$$

$$Re \cong 136.$$

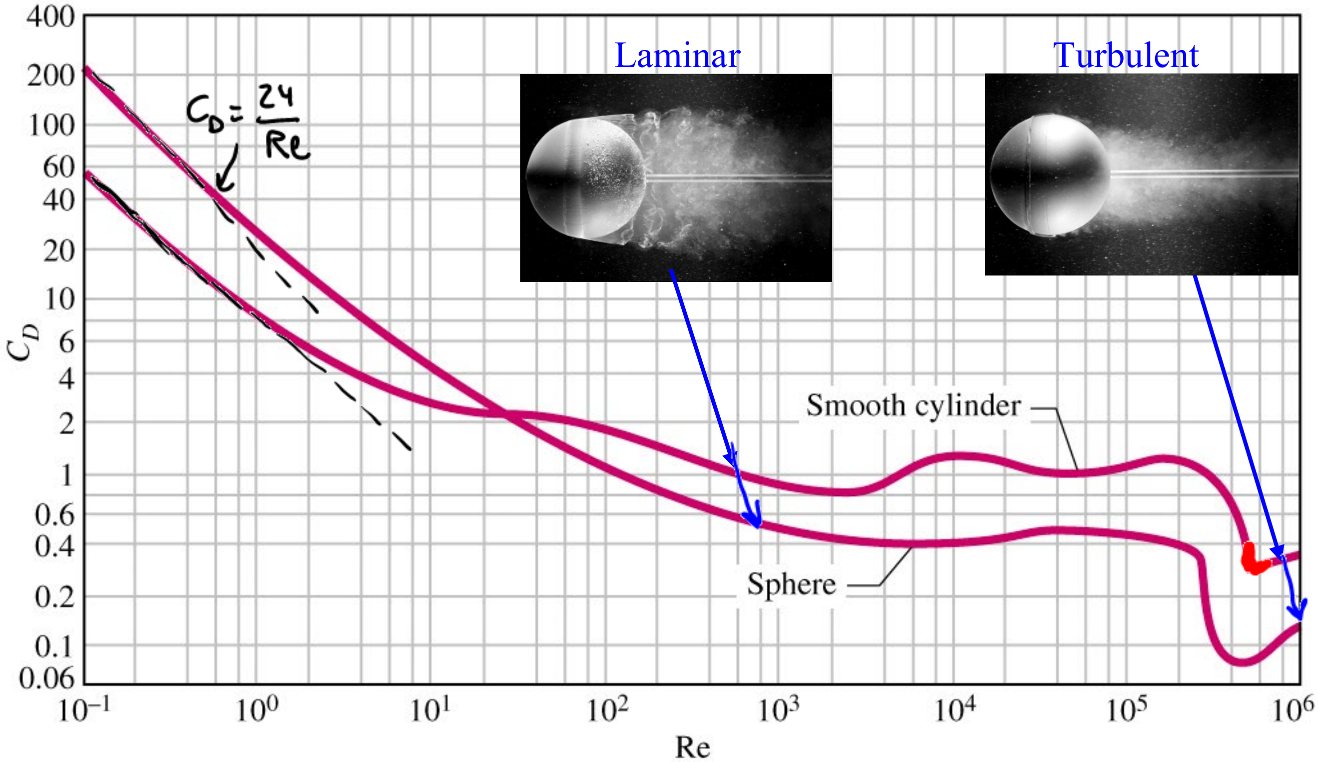
$$C_D \cong 0.90149$$

$$C_D = 0.901$$

Re	C_D
135.869	0.90149

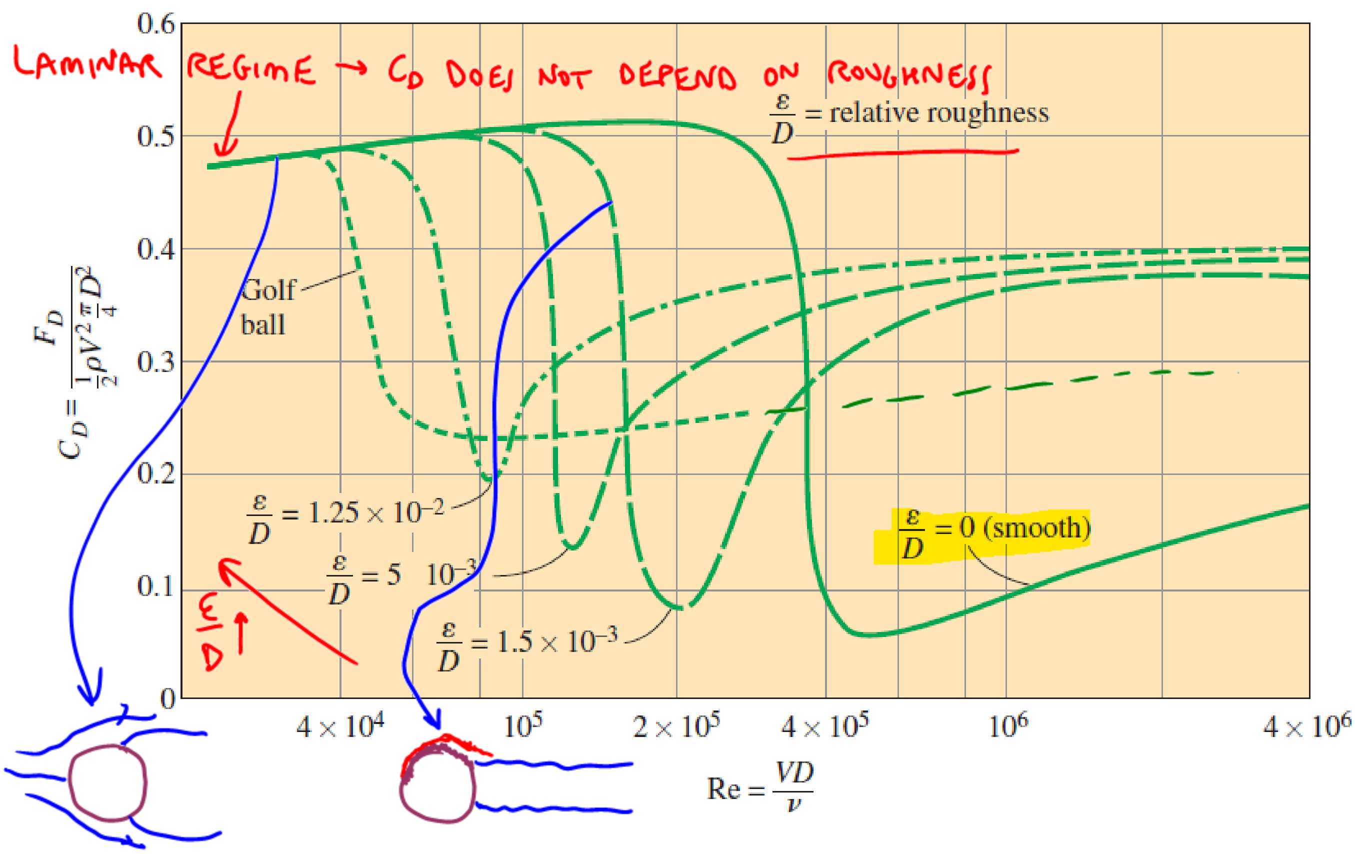
Aerodynamic Drag on Smooth Cylinders

Circular cylinder C_D values also vary with Reynolds number, similarly to those of a sphere. Data are again based on Hermann Schlichting, *Boundary Layer Theory*, 1954.



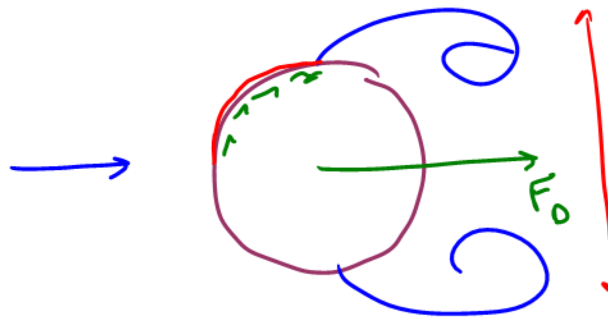
Figures from Çengel and Cimbala, Ed. 4.

The Drag Crisis and the Effect of Roughness on Spheres and Cylinders

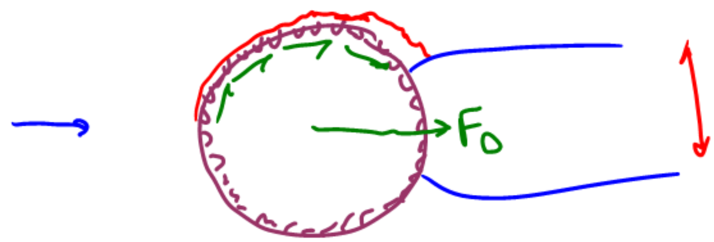


Why do Golf Balls Have Dimples?

- If smooth golf ball, laminar BL



- With dimples, laminar BL is tripped to a turbulent BL



- SKIN FRICTION DRAG IS HIGHER FOR TURBULENT CASE
- PRESSURE DRAG IS MUCH LOWER " "

(Pressure Drag "wins")